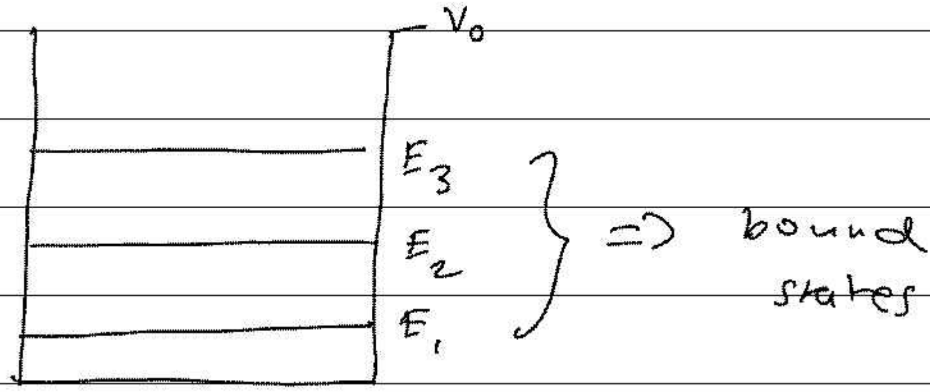
Commutation and Uncertainty Principle



$$[\hat{A}, \hat{B}] = [\hat{A}\hat{B} - \hat{B}\hat{A}] = 0$$

$\hat{A}$ and $\hat{B}$ commute

$$[\hat{T}_x, \hat{p}_x] = [\hat{T}_x \hat{p}_x - \hat{p}_x T_x] f(x)$$

equal

$$\hat{T}_x \hat{p}_x f(x) = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\left[-i\hbar \frac{d}{dx} f(x)\right]$$

$$= \frac{i\hbar^3}{2m}\frac{d^3 f}{dx^3}$$

$$\hat{p}_x \hat{T}_x f(x) = -i\hbar \frac{d}{dx}\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} f(x)\right]$$

$$= \frac{i\hbar^3}{2m}\frac{d^3 f}{dx^3}$$

$\hat{T}_x, \hat{p}_x$ ⇒ ~~observables~~ operators commute

$$[\hat{x}, \hat{p}_x] \equiv [\hat{x}\hat{p}_x - \hat{p}_x\hat{x}] f(x)$$

$$\hat{x}\hat{p}_x f(x) = x\left[-i\hbar \frac{df}{dx}\right] = -i\hbar x \frac{df}{dx}$$

$$\hat{p}_x \hat{x} f(x) = -i\hbar \frac{d}{dx}\left[x f(x)\right]$$

$$= -i\hbar f(x) - i\hbar x \frac{df}{dx}$$

$$[\hat{x}, \hat{p}_x] = i\hbar$$

$$\Delta x \, \Delta p_x \geq \frac{\hbar}{2}$$

Variance

$$\sigma_A^2 = \langle (A - \langle A \rangle)^2 \rangle$$

$$\langle (A - \langle A \rangle)^2 \rangle = \int \psi^* (A - \langle A \rangle)^2 \psi \, dx$$

$$= \int \psi^* \left(\hat{A}^2 - 2\hat{A}\langle A \rangle + \langle A \rangle^2\right) \psi \, dx$$

$$= \int \psi^* \hat{A}^2 dx - \int \psi^* 2\hat{A}\langle A \rangle \psi \, dx + \int \psi^* \langle A \rangle^2 \psi \, dx$$

$$= \langle A^2 \rangle \; \underbrace{- 2\langle A \rangle \int \psi^* \hat{A} \psi \, dx}_{-2\langle A \rangle^2} + \langle A \rangle^2 \underbrace{\int \psi^* \psi \, dx}_{=1}$$

$$\underline{\sigma_A^2 = \langle A^2\rangle - \langle A\rangle^2}$$

$$\sigma_A \sigma_B \geq \frac{1}{2} \left| \int \Psi^* [\hat{A}, \hat{B}] \Psi \, dx \right|$$

$$\sigma_A \sigma_B \geq \frac{1}{2} \left| \langle [\hat{A}, \hat{B}] \rangle \right|$$

$$\sigma_A = \Delta A \qquad \sigma_B = \Delta B$$

$$\sigma_x \sigma_{p_x} \geq \frac{1}{2} \left| \langle [\hat{x}, \hat{p}_x] \rangle \right|$$

$$\int \Psi^* [\hat{x}\hat{p}_x - \hat{p}_x \hat{x}] \Psi \, dx = i\hbar$$

$$\frac{1}{2} |i\hbar| = \frac{\hbar^2}{2}$$

$$\underline{\sigma_x \sigma_{p_x} \geq \frac{\hbar}{2}}$$

$$\sigma_A^2 = \langle A^2\rangle - \langle A\rangle^2$$

$$\sigma_B^2 = \langle B^2\rangle - \langle B\rangle^2$$

$$\underline{\sigma_A \sigma_B \geq \frac{\hbar}{2}}$$

$$\sigma_A = \sigma_x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$$
$$\sigma_B = \sigma_{p_x} = \sqrt{\langle p_x^2\rangle - \langle p_x\rangle^2}$$

do this for a particle in a 1-D box

σ_x:
$$\langle x^2\rangle = \frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}$$

$$\langle x\rangle = \frac{L}{2}$$

$$\sigma_x = \sqrt{\frac{L^2}{3} - \frac{L^2}{2n^2\pi^2} - \left(\frac{L}{2}\right)^2}$$

$$\sigma_x = \frac{L}{2\pi n}\sqrt{\left[\frac{n^2\pi^2}{3} - 2\right]}$$

σ_{p_x}: $\langle p_x\rangle = 0$

$$\langle p_x^2\rangle = 2m\langle E\rangle = 2m\cdot\frac{n^2h^2}{8mL^2}$$
$$= \frac{n^2h^2}{4L^2}$$

$$\sigma_{p_x} = \sqrt{\langle p_x^2\rangle - \langle p_x\rangle^2} = \frac{nh}{2L}$$

$$\sigma_x\sigma_{p_x} = \frac{nh}{2L}\cdot\frac{L}{2\pi n}\sqrt{\left(\frac{n^2\pi^2}{3} - 2\right)}$$
$$= \frac{h}{4\pi}\sqrt{\left(\frac{n^2\pi^2}{3} - 2\right)}$$

$\underline{\underline{n=1}}$ $\sigma_x \sigma_{p_x} = \frac{h}{4\pi}\sqrt{\left(\frac{\pi^2}{3} - 2\right)}$

$= \underline{0.58\hbar}$